

SIMPLE AND ACCURATE EXPRESSION FOR THE DOMINANT MODE
PROPERTIES OF OPEN GROOVE GUIDE

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Groove guide, one of several low-loss waveguides proposed some years ago for use at millimeter wavelengths, is again receiving attention in the literature. A new transverse equivalent network and dispersion relation for the properties of the dominant mode are presented here which are extremely simple in form and yet very accurate. Comparisons with accurate published measurements indicate better agreement with this new theory than with any previous theory.

A. INTRODUCTION

Groove guide is one of a group of low-loss waveguiding structures proposed some years ago for use at millimeter wavelengths. Results for the propagation characteristics of the dominant mode in groove guide have been published previously. There exist theoretical expressions which are simple but approximate, more accurate expressions which involve infinite sums and are messy to compute from, and careful measured results. We present here a new expression for the propagation constant of groove guide, which is very accurate, yet in closed form and simple. The microwave network approach used in the derivation of the new expression is summarized, and then comparisons are made with previously published theoretical and experimental results. It will be seen that the new expression provides excellent agreement with measurement, and in fact better agreement than with any previous theoretical data.

The motivation for obtaining an improved expression for the propagation constant of groove guide, and in the process a transverse equivalent network which is simple and whose constituents are all in closed form, is that groove guide appears to be an excellent low-loss waveguide upon which can be based a number of novel leaky-wave antennas for the millimeter wavelength range. The results of this paper then form an important step in the analysis of such antennas. One antenna in this class has been described recently [1, 2].

The cross section of groove guide is shown in Fig. 1, and an indication of the dominant mode electric field lines present in its cross section is given in Fig. 2(a). One should first note that the structure resembles that of rectangular waveguide with most of its top and bottom walls removed. The groove guide can therefore be excited by

providing a smooth tapered transition between it and a feed rectangular waveguide. Furthermore, if symmetry is maintained, many components can be designed for groove guide which are analogues of those in rectangular guide.

The greater width in the middle, or central, region was shown by T. Nakahara [3-5], the inventor of groove guide, to serve as the mechanism that confines the field in the vertical direction, much as the dielectric central region does in H guide. The field thus decays exponentially away from the central region in the narrower regions above and below, as shown in Fig. 2(b). If the narrower regions are sufficiently long, it does not matter if they remain open or are closed off at the ends.

The theoretical approach to the propagation constant of the dominant mode taken by most of the previous investigators has been to produce a first-order result by taking only the dominant transverse mode in each region of the cross section, and then obtaining the dispersion relation on use of the transverse resonance condition. That procedure, which neglects the presence of all higher transverse modes, is equivalent to accounting for the step junction between the central and outer regions by employing a transformer only, and by ignoring the junction susceptance entirely. With that approximation, a simple dispersion relation is obtained, which produces reasonably good agreement with measured data when the step discontinuity is small. More accurate theoretical phrasings were presented in some references by accounting for the susceptance by taking an infinite number of higher modes on each side of the step junction and then mode matching at the junction. The resulting expressions involve matrices which, even after the necessary truncation, are messy to compute from. When only one or two higher modes are included, the improvement in accuracy is quite small and the added complexity in calculation is substantial.

The approach in this paper is to establish a proper transverse equivalent network, identify the appropriate transverse mode (which is hybrid), obtain an accurate expression in closed form for the step junction susceptance, and then apply the transverse resonance condition to the now-complete transverse equivalent network, which yields the relevant dispersion relation for the propagation constant. This dispersion relation is simple, in

closed form, and very accurate, as demonstrated by comparison with measured data from references 4 and 5.

B. THE TRANSVERSE EQUIVALENT NETWORK

The complete transverse equivalent network for the groove guide is derived by starting with a proper phrasing of the problem and then by putting together all the constituent elements. The essential new constituent in the transverse equivalent network is a simple closed form expression for the step junction susceptance.

To begin with, however, we must identify the correct mode in the y direction (see Fig. 2). We first note that with respect to the z (longitudinal) direction the overall guided mode is a TE (or H) mode; that is, there exists only a component of H in the z direction. This result is to be expected since the groove guide consists of a perfectly conducting outer structure filled with only a single dielectric material (air). In the y direction, however, there exist both E_y and H_y components, so that the mode is hybrid in that direction.

Since the groove guide is uniform in the z direction, and its field has only an H_z component, the hybrid mode in the y direction is seen to be what is called by some an H-type mode with respect to the z direction, and by others an LSE mode with respect to the z direction. We prefer the former notation, and we shall designate the mode in the y direction as an $H^{(z)}$ -type mode. Altschuler and Goldstone [6] discuss such modes in detail and present the field components for them and the characteristic admittances for transmission lines representative of them. For this mode, we find that the characteristic admittance is given by

$$Y_o = \frac{k_o^2 - k_y^2}{\omega \mu k_y} \quad (1)$$

where k_y is the propagation constant of the transmission line.

The step junction is a lossless asymmetric discontinuity, and it therefore requires three real quantities for its characterization. It has been found by experience, however, that for most situations the network conveniently reduces to a shunt network comprised of a shunt susceptance B and a transformer with turns ratio n .

Employing the mode functions for the $H^{(z)}$ -type mode mentioned above, the turns ratio n can be derived in the usual manner to yield

$$n = \left[\frac{a'}{a} \right]^{3/2} \frac{4}{\pi} \frac{\cos \frac{\pi a'}{2a}}{1 - (a'/a)^2} \quad (2)$$

To our knowledge, an expression for the shunt susceptance for the step junction subject to the excitation shown in Fig. 2(a) is not available in the

literature. By a simple additional step, however, we can adapt an available, but not widely known, result to our discontinuity of interest.

The available result is a symmetric discontinuity which is contained in Vol. 8 of the MIT Radiation Laboratory Series [7] and is presented there as an illustration of how Babinet's principle may be used creatively. That result, combined with appropriate stored power considerations, permits us to obtain the following result for the step junction discontinuity susceptance:

$$\frac{B}{Y_o} = 0.55 k_y \frac{2a}{\pi} \cot^2 \frac{\pi a'}{2a} \quad (3)$$

The resulting transverse equivalent network becomes that shown in Fig. 3, where bisection has been employed, and where the network has been placed horizontally for convenience. The expressions for parameters B , n and Y_o (and therefore Y_o') are given respectively by (3), (2), and (1). The form of the network and the expressions for its constituents are seen to be eminently simple, and yet they characterize the structure very accurately.

Once the network in Fig. 3 becomes available, the determination of the dispersion relation for the lowest mode becomes an essentially trivial task. By applying the transverse resonance condition, we obtain

$$\cot k_y \frac{b}{2} = \frac{1}{n} \frac{k_y}{|k_y'|} + k_y 0.55 \frac{2a}{\pi} \cot^2 \frac{\pi a'}{2a} \quad (4)$$

The early first-order solution derived by various authors corresponds precisely to the first two terms in (4). The third term in (4) represents a particularly simple and convenient way to take into account the influence of all the higher modes, which the first-order solution admittedly neglects.

C. NUMERICAL RESULTS: COMPARISON WITH MEASUREMENTS

We next verify the accuracy of these new theoretical results, as embodied in dispersion relation (4) and the simple transverse equivalent network shown in Fig. 3. Toward this end, we present now a comparison between our theoretical numbers and the careful experimental results of Nakahara and Kurauchi [4,5] (referred to below as N-K).

In Fig. 9 of reference 5 and Fig. 10 of reference 4, N-K present the results of careful measurements on a variety of groove guides. They give the measured values of λ_c as a function of a' for groove guides of different cross sections, and they show how these values compare with curves obtained using first-order theory. All of those data, plus our theoretical numbers, are contained in Figs. 4(a) and 4(b) presented here; the first-order theory is represented by dashed lines, our more accurate theory by solid lines, and the measured data as discrete points. The cross sections corresponding to each set of curves are

shown as insets.

It is seen that our theoretical curves agree very well with the measured values in almost all cases. On the other hand, the first-order theoretical values are systematically somewhat below both our theory and the measured data. It appears, therefore, that the first-order theory represents a rather good approximation, considering its simplicity, and that the new theory using (4) is indeed significantly more accurate.

D. REFERENCES

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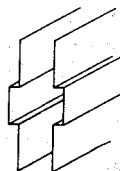


Fig. 1. The open groove guide.

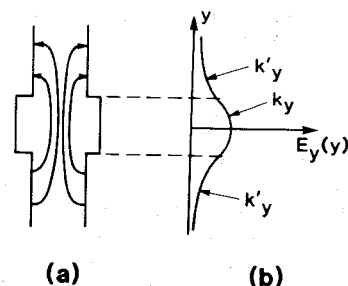


Fig. 2 The electric field of the dominant mode in open groove guide. (a) A sketch of the electric field lines in the cross section, (b) An approximate plot of the vertical component E_y as a function of y , showing that the guided mode is bound transversely to the central grooved region.

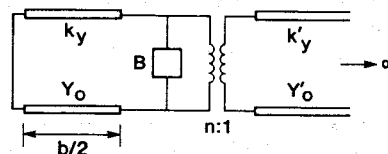


Fig. 3 Complete transverse equivalent network for open groove guide, for the excitation indicated in Fig. 2(a).

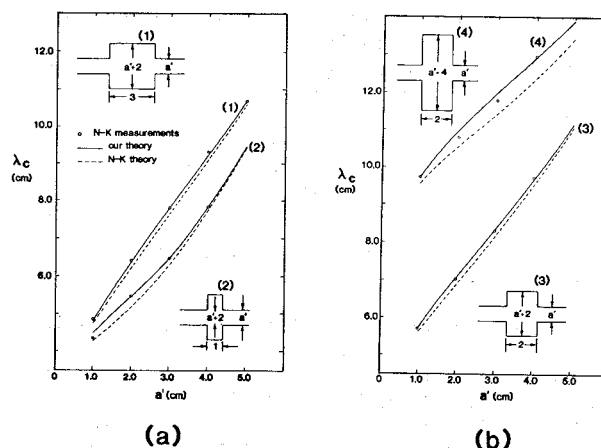


Fig. 4 Comparisons between measured and theoretical values of the cutoff wavelength λ_c for groove guides of various cross sections. The solid lines represent our improved theory, the dashed curves are the first-order theoretical values, and the points are the measured results of N-K [4, 5]. The insets indicate the cross-sectional geometries for each measured point, where the numbers are in cm.